8 QUADRILATERALS

Quadrilaterals, Types and Properties of Quadrilaterals

- **Definition:** A quadrilateral is a simple closed figure with four sides, four angles and four vertices.
- **Types of Quadrilaterals:** There are five types of quadrilaterals:
  (*i*) Parallelogram (*ii*) Rectangle (*iii*) Square (*iv*) Rhombus (*v*) Trapezium
  Kite is also a special type of quadrilateral.
- **Angle Sum Property of a Quadrilateral:** One common property of all quadrilaterals is that the sum of all their angles equals to 360°.
- **Properties of different quadrilaterals:**

S.No.	Property	Parallelogram	Rectangle	Rhombus	Square
1.	All sides are congruent	×	×	✓	✓
2.	Opposite sides are parallel and congruent	✓	✓	✓	✓
3.	All angles are congruent	×	✓	×	✓
4.	Opposite angles are congruent	✓	✓	✓	✓
5.	Diagonals are congruent	×	✓	×	✓
6.	Diagonals are perpendicular	×	×	✓	✓
7.	Diagonals bisect each other	✓	✓	✓	✓
8.	Adjacent angles are supplementary	✓	✓	✓	✓

SOLVED QUESTIONS BASED ON EXERCISE 8.1

Very Short Answer Type Questions [1 Mark]

1. Three angles of quadrilateral are 75°, 90°, 75°. Find the fourth angle. **[NCERT Exemplar]**

Sol. As we know that sum of four angles of quadrilateral is 360°.
Let fourth angle be x.

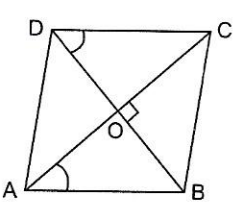
$\therefore \quad 75° + 90° + 75° + x = 360°$

$\Rightarrow \quad x = 360° - 240° = 120°$

$\therefore$ Fourth angle = 120°

2. Diagonals AC and BD of parallelogram ABCD intersect at O. If $\angle BOC = 90°$ and $\angle BDC = 50°$, find $\angle OAB$. **[NCERT Exemplar]**

Sol. In a parallelogram ABCD, O is point of intersection of diagonals AC and BD.



$\angle BDC = \angle DBA$ (Alternate angles as AB || CD)

$\angle BDC = 50°$ (Given)

$\Rightarrow \quad \angle DBA = 50°$

We have $\angle BOC = \angle OBA + \angle OAB$

(Exterior angle is equal to sum of two interior opposite angles)

$\Rightarrow \quad 90° = 50° + \angle OAB$

$\Rightarrow \quad \angle OAB = 90° - 50° = 40°$

3. Can all the angles of a quadrilateral be acute angles? Give reason for your answer. [NCERT Exemplar]

Sol. No, all the angles of quadrilateral cannot be acute angles. If all the angles of quadrilateral will be acute, the sum of all the four angles will be less than 360° which is not possible.

Short Answer Type Questions I [2 Marks]

4. If one angle of a parallelogram is 36° less than twice its adjacent angle, then find the angles of parallelogram. [CBSE 2016]

Sol. Let one angle of parallelogram be x.

Its adjacent angle is $(180° - x)$.

As per question, $x = 2(180 - x) - 36°$

$\Rightarrow$ $x = 360° - 2x - 36°$

$\Rightarrow$ $3x = 324°$

$\Rightarrow$ $x = \frac{324°}{3} = 108°$

$\Rightarrow$ Adjacent angle $= 180° - 108° = 72°$

Hence, the angles of parallelogram are 108°, 72°, 108°, 72°.

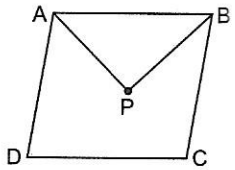
5. In a parallelogram, show that the angle bisectors of two adjacent angles intersect at right angles.

Sol. **Given:** ABCD is a parallelogram such that angle bisectors of adjacent angles A and B intersect at point P.

To prove: $\angle APB = 90°$

Proof: We have $\angle A + \angle B = 180°$ (AD || BC and $\angle A$ and $\angle B$ are consecutive interior angles)

$\frac{1}{2}\angle A + \frac{1}{2}\angle B = 90°$



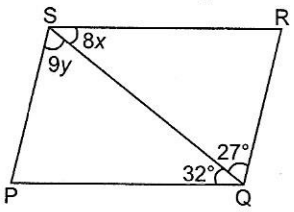
But $\frac{1}{2}\angle A + \frac{1}{2}\angle B + \angle APB = 180°$ (Sum of angles of a triangle is 180°)

$\Rightarrow$ $90° + \angle APB = 180°$

$\Rightarrow$ $\angle APB = 90°$

Hence proved.

6. In the given figure, PQRS is a parallelogram. Find the values of x and y.



Sol. Here, PQRS is a parallelogram.

As PQ || RS,

$\therefore$ $8x = 32°$ (Alternate interior angles)

As PS || QR,

and $9y = 27°$ (Alternate interior angles)

$\Rightarrow$ $x = \frac{32°}{8}$ and $y = \frac{27°}{9}$

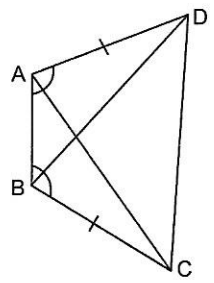
$\Rightarrow$ $x = 4°$ and $y = 3°$

Short Answer Type Questions II [3 Marks]

7. In the given figure, ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$.

Prove that : (*i*) $\Delta ABD \cong \Delta BAC$

(*ii*) BD = AC [CBSE 2010]



Sol. **Given:** ABCD is a quadrilateral in which AD = BC and $\angle DAB = \angle CBA$.

To prove:

(*i*) $\Delta ABD \cong \Delta BAC$

(*ii*) BD = AC

Proof: (*i*) In ΔABD and ΔBAC,

AD = BC (Given)

$\angle DAB = \angle CBA$ (Given)

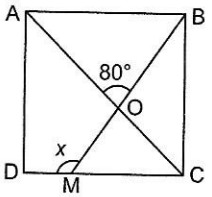
$AB = AB$ (Common)

$\therefore$ $\Delta ABD \cong \Delta BAC$ (SAS congruence rule)

(*ii*) $BD = AC$ (CPCT)

Hence proved.

8. In the given figure, ABCD is a square. A line BM intersects CD at M and diagonal AC at O such that $\angle AOB = 80°$. Find the value of x.



Sol. As diagonal of a square bisects the opposite angles,

$$\angle BAO = \frac{1}{2}\angle BAD = \frac{1}{2} \times 90° = 45°$$

$\angle BAC = \angle ACD$ (Alternate interior angles)

$\therefore$ $\angle ACD = \angle BAC = 45°$ ...(*i*)

Also, $\angle AOB = \angle MOC = 80°$ ...(*ii*) (Vertically opposite angles)

Now, $x = \angle MOC + \angle OCM$ (Exterior angle is equal to sum of two interior opposite angles)

$\therefore$ $x = 80° + 45° = 125°$

Long Answer Type Questions [4 Marks]

9. Two parallel lines l and m are intersected by a transversal p. Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.

Sol. **Given:** BA, BC, DC, DA are bisectors of $\angle PAC$, $\angle QCA$, $\angle ACR$ and $\angle SAC$ respectively.

To prove: ABCD is a rectangle.

Proof: We have $\angle PAC = \angle ACR$

(Alternate interior angles as $l \parallel m$ and p is transversal)

$$\frac{1}{2}\angle PAC = \frac{1}{2}\angle ACR$$

$\Rightarrow$ $\angle BAC = \angle ACD$

(As BA and DC are bisectors of $\angle PAC$ and $\angle ACR$ respectively)

But these are alternate angles. This shows that AB || CD

Similarly, BC || AD

$\Rightarrow$ Quadrilateral ABCD is a parallelogram.

Now, $\angle PAC + \angle CAS = 180°$ ...(*i*) (Linear pair axiom)

$\Rightarrow$ $\frac{1}{2}\angle PAC + \frac{1}{2}\angle CAS = 90° \Rightarrow \angle BAC + \angle CAD = 90° \Rightarrow \angle BAD = 90°$ ...(*ii*)

From (*i*) and (*ii*), we can say that ABCD is a rectangle. Hence proved.

10. ABCD is a rhombus and AB is produced to E and F such that AE = AB = BF. Prove that ED and FC are perpendicular to each other.

Sol. **Given:** ABCD is a rhombus. AB produced to E and F such that AE = AB = BF

Construction: Join ED and CF and produce it to meet at G.

To prove: ED $\perp$ FC

Proof: AB is produced to points E and F such that

$AE = AB = BF$ ...(*i*)

Also, since ABCD is a rhombus

$AB = CD = BC = AD$ ...(*ii*)

Now, in ΔBCF, $BC = BF$ [From (*i*) and (*ii*)]

$\Rightarrow$ $\angle 1 = \angle 2$

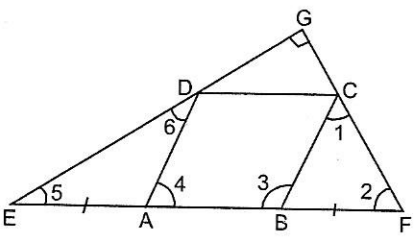
$\angle 3 = \angle 1 + \angle 2$ [Exterior angle]

$\angle 3 = 2\angle 2$ ...(*iii*)

Similarly, $AE = AD$

$\angle 5 = \angle 6$

$\Rightarrow$ $\angle 4 = \angle 5 + \angle 6 = 2\angle 5$ ...(*iv*)



Adding (*iii*) and (*iv*) we get

$$\angle 4 + \angle 3 = 2\angle 5 + 2\angle 2$$

$\Rightarrow$ $180° = 2(\angle 5 + \angle 2)$ [$\because$ $\angle 4$ and $\angle 3$ are consecutive interior angles]

$\Rightarrow$ $\angle 5 + \angle 2 = 90°$

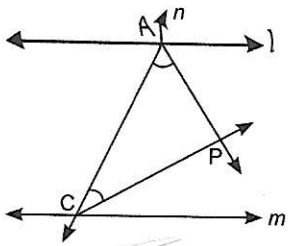
$\therefore$ EG $\perp$ FC. Now in ΔEGF

$\angle 5 + \angle 2 + \angle EGF = 180° \Rightarrow \angle EGF = 90°$

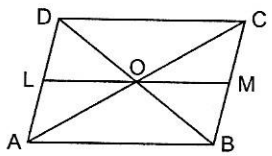
Hence proved.

PRACTICE QUESTIONS BASED ON EXERCISE 8.1

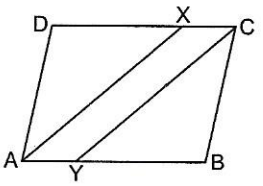
1. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3 : 7 : 6 : 4, then name the type of quadrilateral ABCD. **[NCERT Exemplar]**
2. The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O. If $\angle CAD = 32°$ and $\angle AOB = 70°$, find $\angle DBC$. **[NCERT Exemplar]**
3. Diagonals of a quadrilateral ABCD bisect each other. If $\angle A = 35°$, determine $\angle B$. **[NCERT Exemplar]**
4. In the given figure, AP and CP are bisectors of $\angle A$ and $\angle C$ respectively and $l \,||\, m$. Find the measure of $\angle APC$.



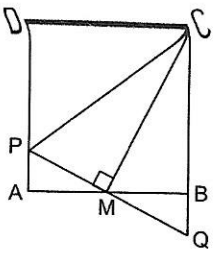
5. In the given figure, ABCD is a parallelogram in which diagonals AC and BD intersect at O. A line segment LM is drawn passing through O. Prove that LO = OM. **[CBSE 2010]**



6. In the given figure, ABCD is a parallelogram and line segments AX and CY bisect the angles A and C respectively. Show that AX || CY.



7. In the given figure, ABCD is a square. M is the mid-point of AB and PQ $\perp$ CM. Prove that CP = CQ. **[CBSE 2010]**



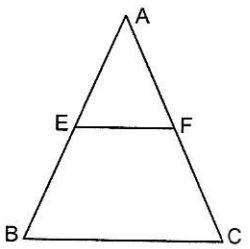
8. In a quadrilateral PQRS, the bisectors of $\angle R$ and $\angle S$ meet at O point T. Show that $\angle P + \angle Q = 2\angle RTS$. **[CBSE 2016]**
9. Prove that a diagonal of a parallelogram divides it into two congruent triangles. **[CBSE 2010]**
10. If two parallelograms PQAD and PQBC are on the opposite sides of PQ, prove that ABCD is a parallelogram.

The Mid-point Theorem

- **The Mid-point Theorem:** The line segment joining the mid-points of two sides of a triangle is parallel to the third side.

  In ΔABC, E and F are mid-points of sides AB and AC respectively.

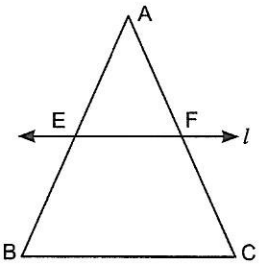
  By mid-point theorem, EF || BC and EF = $\frac{1}{2}$ BC



- **Converse of the Mid-point Theorem:** The line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

  In ΔABC, a line l is parallel to the third side, i.e. BC and E is mid-point of AB.

  By the converse of the mid-point theorem, F is mid-point of AC.

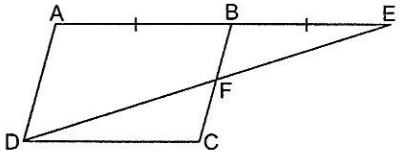


SOLVED QUESTIONS BASED ON EXERCISE 8.2

Short Answer Type Questions I [2 Marks]

1. ABCD is a parallelogram. AB is produced to E so that BE = AB. Prove the ED bisects BC. [CBSE 2010]

Sol. **Given:** ABCD is a parallelogram. AB is produced to E such that BE = AB



To prove: ED bisects BC.

i.e. $BF = FC$

Construction: Join D to E which intersects BC at F.

Proof: We have $AB = DC$ (Opposite sides of parallelogram)

But $AB = BE$ (Given)

$\therefore$ $BE = DC$

In ΔBEF and ΔCDF, $BE = DC$ (Proved above)

$\angle BEF = \angle CDF$ (Alternate interior angles)

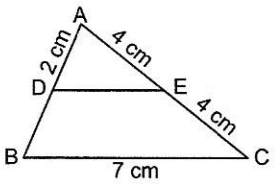
$\angle BFE = \angle CFD$ (Vertically opposite angles)

$\therefore$ $\Delta BEF \cong \Delta CDF$ (AAS congruence rule)

$\therefore$ $BF = FC$ (CPCT)

$\therefore$ ED bisects BC. Hence proved.

2. In the given figure, DE || BC. Find BD.



Sol. As AE = EC = 4 cm,

$\therefore$ E is mid-point of AC.

Also, $DE \parallel BC$ (Given)

$\therefore$ By the converse of mid-point theorem, we have

D is mid-point of AB

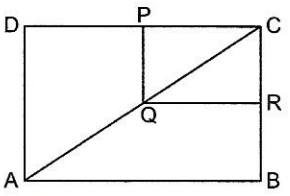
$\Rightarrow$ $AD = BD$

$\Rightarrow$ $BD = 2$ cm (As given AD = 2 cm)

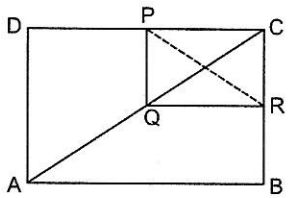
Short Answer Type Questions II [3 Marks]

3. In the given figure, ABCD and PQRC are rectangles and Q is the mid-point of AC. Prove that:

(i) DP = PC **(ii) PR = $\frac{1}{2}$AC** [CBSE 2010]



Sol. **Given:** ABCD and PQRC are two rectangles and Q is the mid-point of AC.



To prove: (*i*) DP = PC (*ii*) PR = $\frac{1}{2}$ AC

Proof: (*i*) In ΔACD,

Q is the mid-point of AC

$\angle ADC = \angle QPC = 90°$ (Each angle of rectangle is right angle)

But these are corresponding angles.

$\Rightarrow$ PQ || DA

$\therefore$ P is the mid-point of CD.

i.e. DP = PC

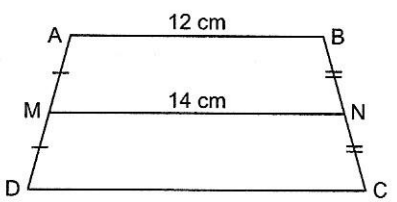
(*ii*) We have QC = PR (Diagonals of rectangle are equal)

and QC = $\frac{1}{2}$ AC (Given)

$\therefore$ PR = $\frac{1}{2}$ AC Hence proved.

4. ABCD is a trapezium in which AB || DC. M and N are the mid-points of AD and BC respectively. If AB = 12 cm and MN = 14 cm, find CD. **[HOTS]**

Sol. Here, ABCD is a trapezium in which, AB || DC and M and N are mid-points of AD and BC respectively.



Since the line segment joining the mid-points of non-parallel sides of trapezium is half of the sum of the lengths of its parallel sides,

$\Rightarrow$ MN = $\frac{1}{2}$ (AB + CD)

$\Rightarrow$ 14 = $\frac{1}{2}$ (12 + CD)

$\Rightarrow$ 28 = 12 + CD

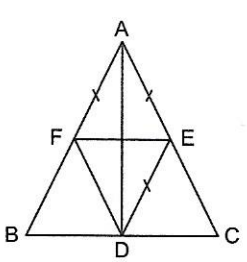
$\Rightarrow$ CD = 28 – 12 = 16 cm

Long Answer Type Questions [4 Marks]

5. In ΔABC is isosceles with AB = AC. D, E and F are the mid-points of sides BC, CA and AB respectively. Show that the line segment AD is perpendicular to the line segment EF and is bisected by it.

Sol. **Given:** ΔABC is isosceles with AB = AC. D, E and F are the mid-points of BC, CA and AB respectively.

To prove: AD ⊥ EF and is bisected by it.



Construction: Join D, E and F and AD.

Proof: We have DE || AB and DE = $\frac{1}{2}$ AB ...(*i*)

and DF || AC and DF = $\frac{1}{2}$ AC ...(*ii*)

(Line segment joining mid-points of two sides of a triangle is parallel to the third side and is half of it.)

$AB = AC$ ...(*iii*)

$\therefore$ $AF = \frac{1}{2}AB$, $AE = \frac{1}{2}AC$ ...(*iv*)

From (*i*), (*ii*), (*iii*) and (*iv*), we get $DE = DF = AF = AE$

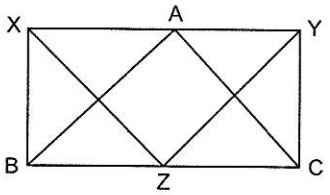
and also, $DF \parallel AE$ and $DE \parallel AF$

$\Rightarrow$ DEAF is a rhombus.

Since diagonals of a rhombus bisect each other at right angles,

$\therefore$ $AD \perp EF$ and is bisected by it.

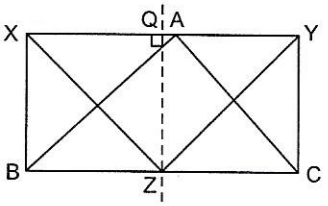
6. In the given figure, BX and CY are perpendiculars to a line through the vertex A of ΔABC and Z is the mid-point of BC. Prove that XZ = YZ. **[HOTS, CBSE 2015]**



Sol. Given: BX and CY are perpendiculars to a line XAY.

To prove: XZ = YZ

Construction: Draw $ZQ \perp XY$.



Proof: We have $BX \perp XY$ (Given)

and $CY \perp XY$ (Given)

Also, $ZQ \perp XY$ (By construction)

$\therefore$ $BX \parallel ZQ \parallel CY$ (Perpendiculars on same line are parallel to each other)

Now, as $BX \parallel ZQ \parallel CY$

and Z is mid-point of BC (Given)

$\therefore$ By mid-point theorem, we have Q is mid-point of XY.

In ΔXQZ and ΔYQZ,

$XQ = QY$ (Q is mid-point of XY)

$\angle XQZ = \angle YQZ = 90°$ (By construction)

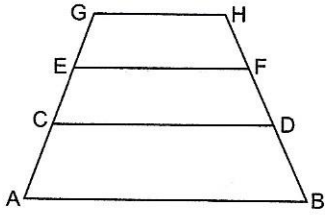
$QZ = QZ$ (Common)

$\therefore$ $\Delta XQZ \cong \Delta YQZ$ (SAS congruence rule)

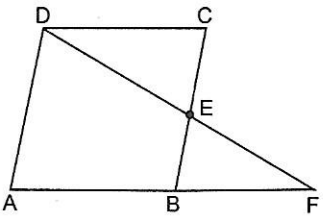
$\Rightarrow$ $XZ = YZ$ (CPCT) Hence proved.

PRACTICE QUESTIONS BASED ON EXERCISE 8.2

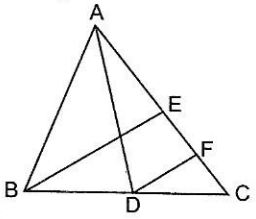
1. D, E and F are respectively the mid-points of sides BC, CA and AB of an equilateral ΔABC. Prove that ΔDEF is also an equilateral triangle.
2. In the given figure, $AB \parallel CD \parallel EF \parallel GH$ and HF = FD = DB. If AC = 1.5 cm, find AG.



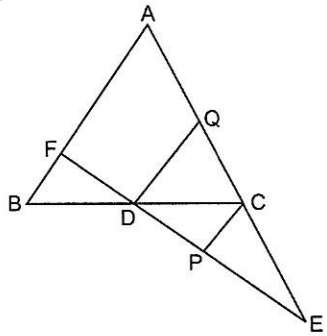
3. ABCD is a parallelogram and E is the mid-point of side BC. DE and AB on producing meet at F. Prove that AF = 2AB.



4. In the given figure, AD and BE are medians of ΔABC and $BE \parallel DF$. Prove that $CF = \frac{1}{4}AC$.



5. In the given figure, the side AC of ΔABC is produced to E such that $CE = \frac{1}{2}$ AC. If D is the mid-point of BC and ED is produced to meet AB at F and CP and DQ are drawn parallel to BA, then prove that $FD = \frac{1}{3}$ FE.



6. Show that the quadrilateral formed by joining the mid-points of the sides of a square, is also a square.

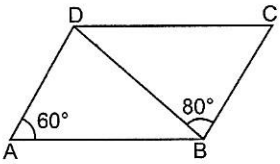
Value Based Questions

1. Four students of your class of different faith Hindu, Muslim, Sikh and Christian bond a human chain like a quadrilateral of sides 12 cm, 5 cm, 12 cm, 15 cm and angle between the sides 12 cm and 5 cm is 90°.
   (*i*) Name the type of quadrilateral.
   (*ii*) What assumption will be arrived from this activity?
2. A resident welfare society has developed a land having parallelogram shape of side 12 cm each and pair of opposite sides are parallel. They divided the land into five parts by joining the mid-points of consecutive sides of given quadrilateral. Here, one part quadrilateral and another four parts are triangles. In one middle part, they developed gardening and in rest of the part, they developed classical musical centres.
   (*i*) Name the quadrilateral formed in middle.
   (*ii*) What conclusion you will draw through this activity?
3. A religious place is in the shape of an equilateral triangle out of which a triangular park is developed inside it by joining the mid-points of consecutive sides for charitable dispensary purpose.
   (*i*) Name the shape of charitable dispensary.
   (*ii*) What conclusion you will derive from this activity?

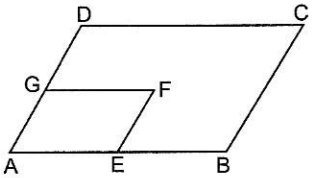
INTEGRATED EXERCISE

Very Short Answer Type Questions [1 Mark]

1. In a parallelogram ABCD of the given figure, $\angle DAB = 60°$ and $\angle DBC = 80°$. Find $\angle ABD$. **[CBSE 2014]**

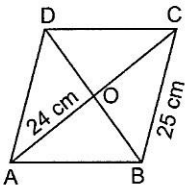


2. In the given figure, ABCD and AEFG are two parallelograms. If $\angle C = 55°$ determine $\angle F$. **[NCERT Exemplar]**



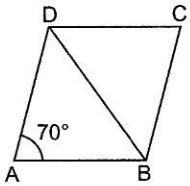
Short Answer Type Questions I [2 Marks]

3. Two angles of a quadrilateral are 50° and 80° and other two angles are in the ratio 8 : 15. Find the remaining two angles.
4. ABCD is a parallelogram. If $\angle A = 65°$, find $\angle B + \angle D$.
5. ABCD is a rectangle with $\angle BAC = 42°$. Determine $\angle DBC$.
6. In the given figure, ABCD is a rhombus in which BC = 25 cm and AO = 24 cm. Find the sum of the lengths of the diagonals. **[CBSE 2010]**

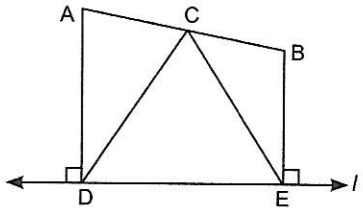


Short Answer Type Questions II [3 Marks]

7. ABCD is a rhombus such that $\angle ACB = 40°$. Find $\angle ADB$. **[NCERT Exemplar]**
8. In the given figure, ABCD is a rhombus. If $\angle DAB = 70°$, then find $\angle CDB$. **[CBSE 2010]**



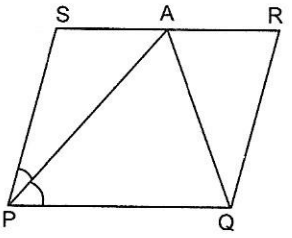
9. In figure, A and B are on the same side of a line l. $AD \perp l$ and $BE \perp l$. If C is the mid-point of AB, prove that CD = CE.



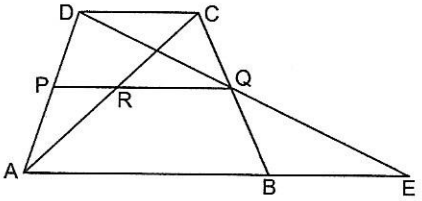
10. Prove that the straight line joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of the trapezium and is equal to half of their difference.
11. P and Q are points of trisection of the diagonal BD of a parallelogram ABCD. Prove that CQ is parallel to AP and AC bisects PQ.

Long Answer Type Questions [4 Marks]

12. In a trapezium ABCD, AB || CD and AD = BC. If $\angle D = 70°$, find $\angle C$.
13. In the given figure, PQRS is a parallelogram and $\angle SPQ = 60°$. If the bisectors of $\angle P$ and $\angle Q$ meet at A on RS, prove that A is the mid-point of RS.



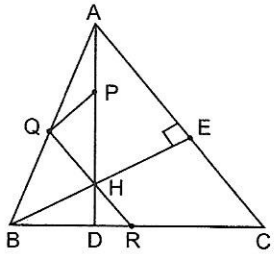
14. Prove that an isosceles trapezium is always cyclic and its diagonal are equal. **[CBSE 2016]**
15. In the given figure, ABCD is a trapezium in which AB || DC. P and Q are the mid-points of AD and BC respectively. DQ and AB when produced meet at E. Prove that



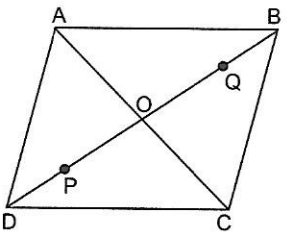
(*i*) DQ = QE, (*ii*) AR = CR,
(*iii*) PR || AB.

ASSESS YOURSELF

1. Can the angles 110°, 80°, 70° and 95° be the angles of a parallelogram? Why or why not? **[NCERT Exemplar]**
2. Can all the angles of a quadrilateral be right angles? Give reason for your answer. **[NCERT Exemplar]**
3. The angles of a quadrilateral are in the ratio 1 : 2 : 4 : 5. Find all the angles of the quadrilateral. **[CBSE 2010]**
4. PQRS is a rhombus with $\angle PQR = 58°$. Determine $\angle PRS$.
5. ABCD is a square. Determine $\angle DCA$.
6. ΔABC and ΔDEF are two triangles such that AB, BC are respectively equal and parallel to DE and EF. Show that AC is equal and parallel to DF.
7. In the given figure, $BE \perp AC$. AD is any line from A to BC intersecting BE at H. P, Q and R are mid-points of AH, AB and BC respectively. Prove that $\angle PQR = 90°$.



8. In a parallelogram PQRS, diagonals PR and QS intersect at O and PR = 6.4 cm and QS = 5.8 cm. Find the measurement of OP and OQ.
9. In a parallelogram ABCD, two points P and Q are taken on its diagonal BD such that DP = BQ. Prove that PQ and AC bisect each other.



10. In a ΔPQR, median PM is produced to X such that PM = MX. Prove that PQXR is a parallelogram.